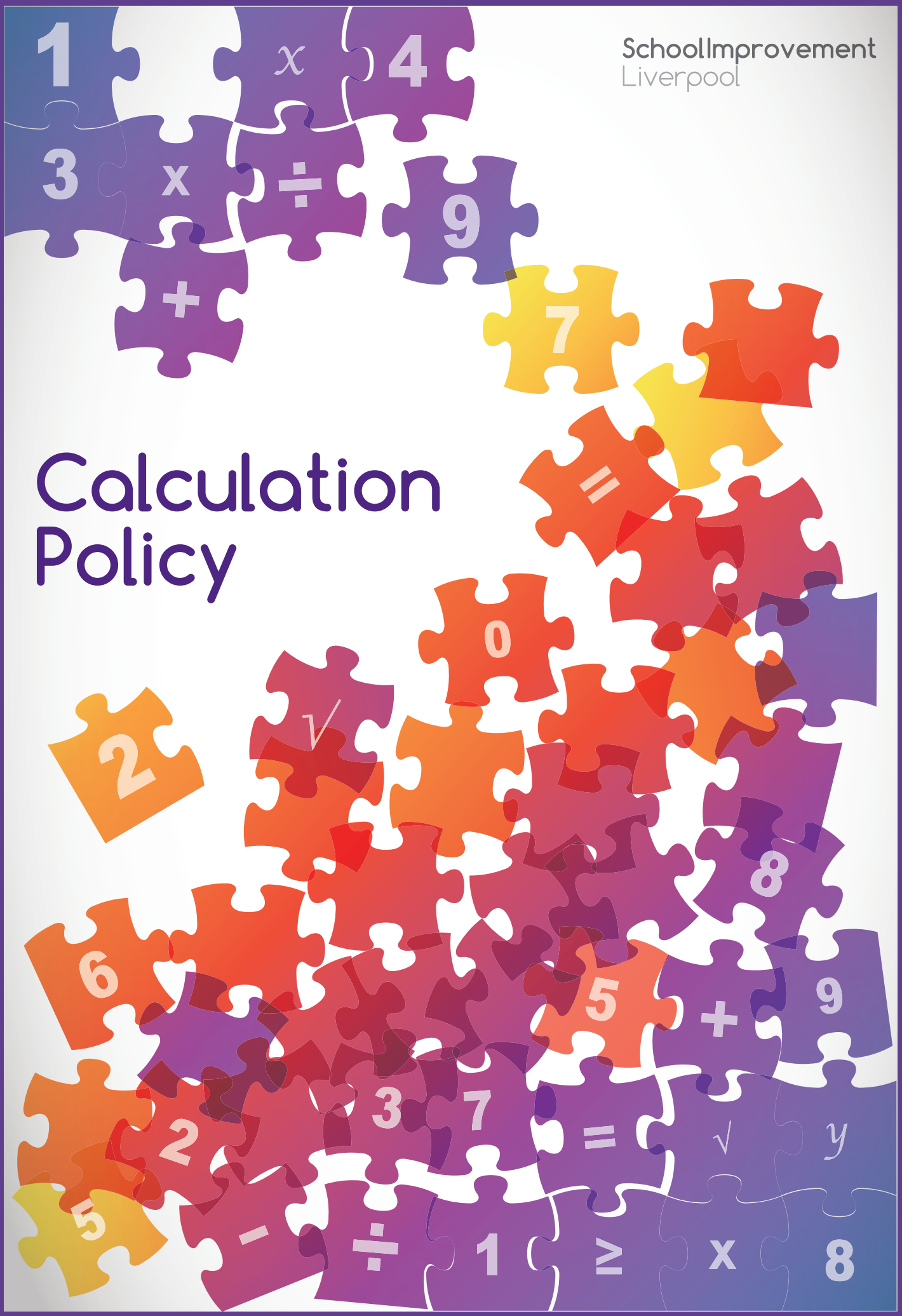
SchoolImprovement
Liverpool

Calculation Policy



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Introduction

The Liverpool Maths Team has developed a calculation policy to support effective implementation of the 2013 Primary National Curriculum.

The policy focuses on the four operations of addition, subtraction, multiplication and division and includes a list of the key mental maths skills that support written methods.

For each operation, there are four stages, starting with the practical methods that support conceptual understanding moving through to methods that allow children to demonstrate efficiency in procedural approaches.

It is important to emphasise that alternative methods may be more appropriate for certain calculations and that informal methods currently used successfully in schools may continue to be used as they support the raised expectations in calculation outlined in this policy.

Addition

Written methods for addition

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of addition.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for addition which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for addition, each stage building towards a more refined method.

There are some key basic skills that children need to help with addition, which include:

- counting
- estimating
- recalling all addition **pairs** to 10, 20 and 100 (7 + 3 = 10, 17 + 3 = 20, 70 + 30 = 100)
- knowing number **facts** to 10 (6 + 2 = 8)
- adding mentally a series of one-digit numbers (5 + 8 + 4)
- adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)
- understanding and using addition and subtraction as inverse operations

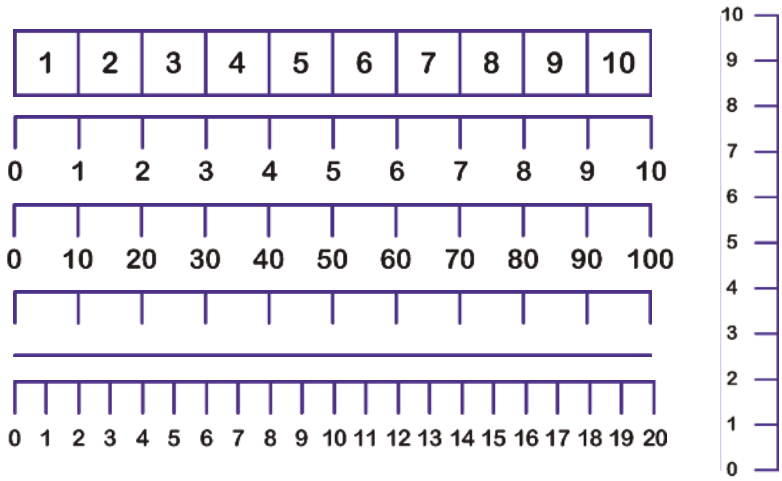
Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations

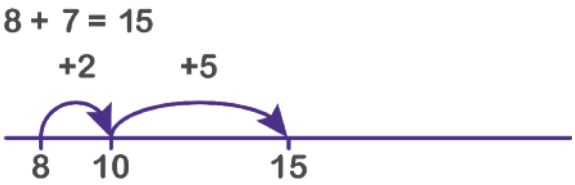
Stage 1: Practical (combining) and adding on (increasing)

Prior to recording addition steps on a number line, children will work practically with equipment where they are **combining** sets of objects. As they become more confident, this practical addition of sets of objects will be mirrored on a number line so that the two are being done together and children are **adding on**. This will prepare them for the abstract concept of adding numbers rather than objects.

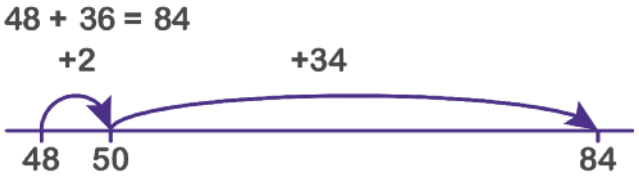
Stage 2: Number tracks and number lines



Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.



In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient



or

In these examples, the 6 in 36 has been partitioned into 2 and 4 which makes bridging through 10 more efficient

With practice, children will need to record fewer jumps

Stage 3: Partitioning (expanded columnar method)

Partition both numbers into tens and units or hundreds, tens and units (using a grid makes this easier)..

48 + 36 = 84

	40	8	
+	30	6	
	70	$^{1}4$	84

148 + 36 = 184

	100	40	8	
+		30	6	
	100	70	$^{1}4$	184

This builds on children's mental maths skills of partitioning and recombining 40 + 30 = 70

8 + 6 = 14

48 + 36 = 84

Stage 4: Efficient (column method)

$$\begin{array}{r} 48 \\ +\ 36 \\ \hline 84 \\ \hline {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 148 \\ +\ 36 \\ \hline 184 \\ \hline {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 48.56 \\ +\ 32.23 \\ \hline 80.79 \\ \hline {\scriptstyle 1} \end{array}$$

Children should be encouraged to estimate their answers first

Column addition remains efficient when used with larger whole numbers or decimals, and when adding more than two numbers, once learned, the method is quick and reliable.

Subtraction

Written methods for Subtraction

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of subtraction.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for subtraction, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- recalling all addition **pairs** to 10, 20 and 100 along with their inverses (7 + 3 = 10, 10 – 3 = 7, 17 + 3 = 20, 20 – 3 = 17, 70 + 30 = 100, 100 – 30 = 70)
- knowing number **facts** to 10 and their inverses (6 + 2 = 8, 8 - 2 = 6)
- subtracting multiples of 10 (160 - 70) using the related subtraction fact, 16 - 7, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)
- understanding and using subtraction and addition as inverse operations

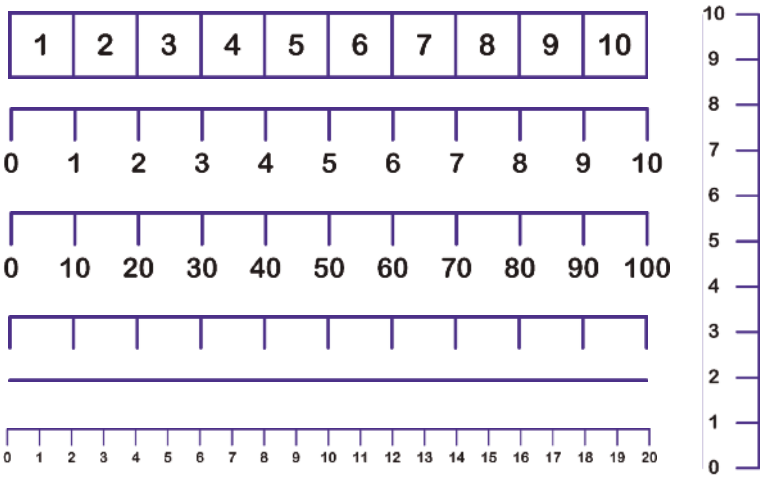
Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations

Stage 1: Practical (taking away)

Prior to recording subtraction steps on a number line, children will work practically with equipment where they are 'taking away' a small group from a larger set of objects. As they become more confident, this practical subtraction will be mirrored on a number line so that the two are being done together. This will prepare them for the abstract concept of subtracting numbers rather than objects.

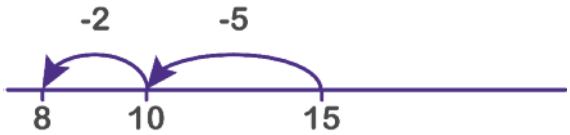
Stage 2: Number tracks and number lines



Counting back (to be introduced before counting up)

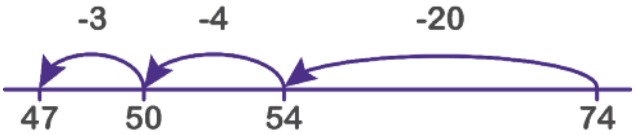
Steps in subtraction can be recorded from right to left on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.

15 - 7 = 8

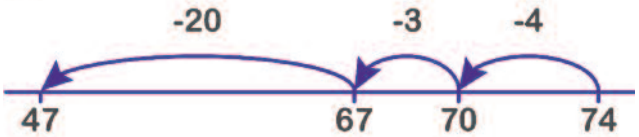


In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient

74 - 27 = 47



or



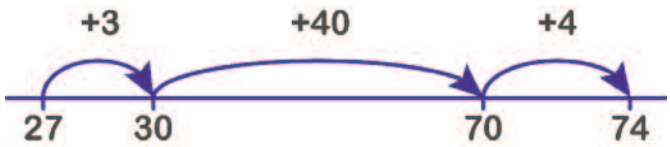
In these examples, 27 has been partitioned into tens and units then the 7 in 27 has been partitioned into 3 and 4 which makes bridging through 10 more efficient

174 - 27 = 147

With practice, children will need to record fewer jumps.

Counting up (to be introduced after counting back)

Steps in subtraction can be recorded from left to right on a number line. The steps often bridge through a multiple of 10.



or

When carrying out money calculations that involve finding change or when calculating time duration, children should use this method

With practice, children will need to record fewer jumps.

They will decide whether to count back or forwards, seeing both as 'finding the difference'. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12 or 86 – 77.

Stage 3: Partitioning (expanded columnar method)

Partition both numbers into tens and units or hundreds, tens and units (using a grid makes this easier).

	60 ~~70~~	1 4	
-	20	7	
	40	7	47

	100	60 ~~70~~	1 4	
		20	7	
	100	40	7	147

Stage 4: Efficient (column method)

6 1 ~~7~~4	6 1 1~~7~~4	48.56
- 27	- 27	- 32.23
47	147	16.33

Children should be encouraged to estimate their answers first

Column subtraction remains efficient when used with larger whole numbers or decimals, once learned, the method is quick and reliable.

Multiplication

Written methods for Multiplication

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of multiplication.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for multiplication, each stage building towards a more refined method.

There are some key basic skills that children need to help with multiplication, which include:

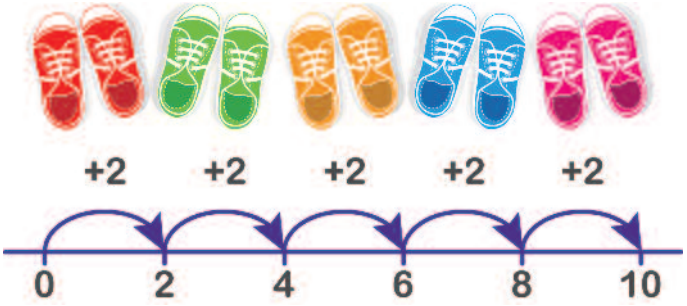
- counting
- estimating
- understanding multiplication as repeated addition
- recalling all multiplication facts to 12 × 12
- partitioning numbers into multiples of one hundred, ten and one
- working out products (70 × 5, 70 × 50, 700 × 5, 700 × 50) using the related fact 7 × 5 and their knowledge of place value
- adding two or more single-digit numbers mentally
- adding multiples of 10 (60 + 70) or of 100 (600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- adding combinations of whole numbers
- understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations

Stage 1: Practical (repeated addition)

Children will work practically with equipment grouping objects to see multiplication as repeated addition. As they become more confident, this practical grouping of objects will be mirrored on a number line using the vocabulary 'lots of', 'groups of', 'how many lots', 'how many times' so that the two are being done together. This will prepare them for the abstract concept of multiplying numbers rather than objects.

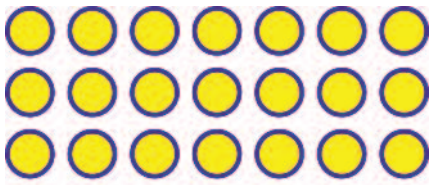


This image can be expressed as:

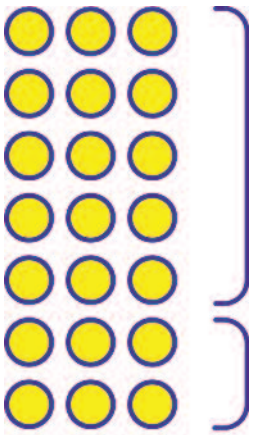
- 2 multiplied by 5
- two, five times
- 5 groups of 2
- 5 lots of 2
- 5 jumps of 2 on a number line

Stage 2: Practical and pictorial arrays (towards grid method)

Children use arrays to demonstrate their understanding of commutativity for multiplication facts



7 x 3 = 21



3 x 7 = 21

Children use their knowledge of known multiplication tables

This 3 x 7 array can also be seen as 3 x 5 add 3 x 2

Stage 3: Partitioning (grid method)

24 x 3 = 72

x	20	4	
3	60	12	72

24 x 32 = 768

x	20	4	
30	600	120	720
2	40	8	48
			768

Stage 4: Short (column)

24 x 3 = 72

```
   24
x   3
   72
    1
```

1241 x 3 = 3723

```
 1241
x    3
 3723
    1
```

Stage 5: Long (column)

24 x 32 = 768

```
   24
 x 32
   48
  720
   1
  765
```

1245 x 13

```
  1245
x   13
  3735
 12450
 16185
```

In the examples given, it is also correct to multiply starting with the tens digit (ie multiplying by the most significant digit first)

Division

Written methods for Division

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of division.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for division which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for division, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- understanding division as repeated subtraction
- partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (432 into 400 + 30 + 2 and also into 300 + 120 + 12)
- recalling multiplication and division facts to 12 × 12
- recognising multiples of one-digit numbers and dividing multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
- knowing how to find a remainder working mentally, for example, find the remainder when 48 is divided by 5
- understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations

Stage 1: Practical (sharing)

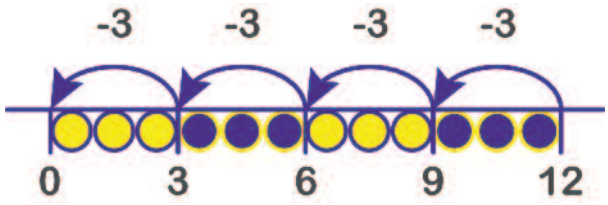
Children will work practically with equipment sharing objects one to one.

12 cakes are shared equally between 3 people.

Stage 2: Number lines (grouping)

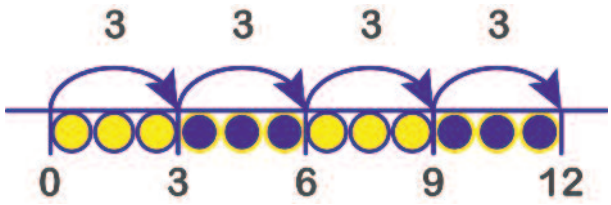
Children will move from sharing objects practically to grouping them, this will be mirrored on a number line, working from right to left so that they see division as repeated subtraction. This will prepare them for the abstract concept of dividing numbers rather than objects.

Each cake box holds 3 cakes, if I have 12 cakes, how many cake boxes will I need?



How many times can I subtract 3 from 12?

Using their knowledge of the inverse relationship between multiplication and division, children can use their multiplication tables when grouping on a number line, working from left to right.



How many groups of 3 are there in 12?

First without and then with remainders and ensuring that divisors offer an appropriate level of challenge.

Stage 3: Short division

372 ÷ 3 = 124 432 ÷ 15 = 28 r12

```
   124
3 | 372
    1
```

```
    28 r12
15 | 432
    13
```

```
    28 12/15
15 | 432
    13
```

remainder as a fraction

```
    28 4/5
15 | 432
    13
```

```
    28.8
15 | 432
    13
```

remainder as a decimal

Stage 4: Long division

560 ÷ 24 = 23 r8 432 ÷ 15 = 28 r12

```
    23 r8
24 | 560
     48
     80
     72
      8
```

```
    28 r12
15 | 432
     300   15x20
     132
     120   15x8
      12
```

```
    28.8
15 | 432.0
     30
     132
     120
      12 0
      12 0
         0
```

(12 ÷ 15 = 0.8)

remainder as a decimal

```
    28 4/5
15 | 432
     30
     132
     120
      12
      12
```

(0.8 = $\frac{4}{5}$)

remainder as a fraction

With long division, there is the opportunity to teach an expanded method first (ie chunking)

The Calculation Sequence – applying the skills

The Sequence	Prompts	Planning
Provide an estimate for the calculation	Using knowledge of number and the number system, rounding and approximating, make a reasonable estimate.	
Teach the calculation skill	What is the objective you are teaching? Include example questions, increasing in complexity, for both operations.	
Ensure you have taught the inverse	Plan example questions, increasing in complexity. Ensure methods used are in line with school calculation policy. Check that children understand that inverse can also be used to check calculations	
Devise similar calculations but include units	Which units do you need to include? Check the measures applicable to your year group for length, weight, capacity, money and time.	
Complete missing box questions	Include units in these questions as above. The box may cover single digits or an entire number. Vary the position of the missing box within the calculation.	
Complete word problems, 1 and 2 step, including units	Write problems, ensuring the numbers are sized correctly in line with the objective and that units are also used.	
Provide opportunities for open ended investigations	Plan example questions and investigations. Ensure children are working with the correct operations, appropriate size of numbers and use of units for context.	

Progression across the year groups
Addition

	Typical calculations	Suitable methods
Y1	U+U TU + U (to 20 including zero)	Practical Number line
Y2	TU + U TU + multiples of 10 TU + TU U + U + U	Practical Number line Expanded columnar
Y3	HTU + U HTU + TU HTU + HTU	Number line Expanded columnar Column
Y4	THTU + HTU THTU + THTU	Expanded columnar Column
Y5	THTU.t + THTU.t THTU.th + THTU.th	Expanded columnar Column
Y6	THTU.tht + THTU.tht	Column

Progression across the year groups
Subtraction

	Typical calculations	Suitable methods
Y1	U-U TU - U (to 20 including zero)	Practical Number line
Y2	TU - U TU - multiples of 10 TU - TU U - U - U	Practical Number line Expanded columnar
Y3	HTU - U HTU – TU HTU - HTU	Number line Expanded columnar Column
Y4	THTU - HTU THTU - THTU	Expanded columnar Column
Y5	THTU.t - THTU.t THTU.th - THTU.th	Expanded columnar Column
Y6	THTU.tht - THTU.tht	Column

Progression across the year groups
Multiplication

	Typical calculations	Suitable methods
Y1	U x U	Practical (repeated addition) Practical and pictorial arrays
Y2	U x U	Practical (repeated addition) Practical and pictorial arrays
Y3	TU x U	Grouping on a number line progressing into Expanded (grid) and into Short
Y4	TU x U HTU x U	Expanded (grid) progressing into Short
Y5	HTU x U THTU x U TU x TU	Expanded (grid) progressing into Short Expanded (grid) progressing into Long
Y6	THTU x U TU x TU HTU x TU THTU x TU U.t x U U.th x U U.t x TU U.t x TU	Short Expanded (grid) progressing into Long Long Expanded (grid) progressing into Short Expanded (grid) progressing into Long

Progression across the year groups
Division

	Typical calculations	Suitable methods
Y1	U ÷ U TU ÷ U	Practical sharing Number-line grouping
Y2	U ÷ U TU ÷ U	Practical sharing Number-line grouping
Y3	TU ÷ U	Grouping on a number line progressing into Short
Y4	TU ÷ U HTU ÷ U	Grouping on a number line progressing into Short Short (remainders to be expressed as r)
Y5	HTU ÷ U THTU ÷ U	Short (remainders to be expressed as r, then as a fraction and as a decimal)
Y6	THTU ÷ U HTU ÷ TU THTU ÷ TU U.th ÷ U TU.th ÷ U HTU.th ÷ U THTU.th ÷ U	Short (remainders to be expressed as r, then as a fraction and as a decimal) Long (remainders to be expressed as r, then as a fraction and as a decimal) Short (remainders to be expressed as a decimal)

Notes

Notes

For more information please contact:

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